Geometry of Singularities and Differential Equations

Celebrating the contributions of Felipe Cano to the Theory of Singularities

Santander, 26th-30th June 2017

ABSTRACTS

SCHEDULE

Monday, June 26	
8:30 – 9:30	Registration Facultad de Ciencias, Universidad de Cantabria
9:30 – 10:15	Frank Loray *Neighborhoods of curves in complex surfaces*
10:15 – 11:00	Jorge Vitorio Pereira *Algebraic integration in bounded genus*
11:00 – 11:30	Coffee Break
11:30 – 12:15	Javier Ribón *Intersections in holomorphic dynamical systems*
12:15 – 13:00	Yohann Genzmer *Dimension of the moduli space of a plane branch in* $\mathbb{C}^2$
13:00 – 15:30	Lunch
15:30 – 16:15	Lorena López *Stable manifolds of diffeomorphisms asymptotic to formal curves*
16:15 – 17:00	Clemen Alonso *Stratification of the dynamics of real vector fields in dimension three*
17:00 – 17:45	Rudy Rosas *Characteristic curves of holomorphic foliations*
18:00 – 20:00	INAGURATION

Tuesday, June 27	
9:30 – 10:15	Vicent Cossart *Ridge of a tangent cone: a forgotten theorem*
10:15 – 11:00	Orlando Villamayor *On the behavior of the multiplicity on points of a variety*
11:00 – 11:30	Coffee Break
11:30 – 12:15	Lê Dũng Tráng *A remark about complex polynomial functions*
12:15 – 13:00	Miguel Fernández *Local uniformization of codimension one foliations*
13:00 – 15:30	Lunch
15:30 – 16:15	Marcio Soares *Khanedani-Suwa variational residues for invariant currents*
16:15 – 17:00	Daniel Panazzolo *Resolution of singularities for differential operators*
17:00 – 17:30	Coffee Break
17:30 – 18:15	André Belotto *The Sard conjecture on Martinet surfaces*

Wednesday, June 28	
9:30 – 10:15	Sergei Yakovenko *Normalization and factorization of linear ordinary differential operators*
10:15 – 11:00	Adam Parusiński *Zariski equisingularity and Whitney conditions*
11:00 – 11:30	Coffee Break
11:30 – 12:15	Fernando Alcalde *Minimal foliations of hyperbolic 3-manifolds*
12:15 – 13:00	Nuria Corral *Jacobian curves of singular foliations*
13:00 – 15:30	Lunch
15:30 – 16:15	Olivier Le Gal *Realization of formal invariant curve*
16:15 – 17:00	Jean-Marie Lion *Ensembles pfaffiens*
17:00 – 17:30	Coffee Break
17:30 – 18:15	Fernando Sanz *Real analytic vector fields with first integral and separatrices*

Thursday, June 29	
9:30 – 10:15	Emmanuel Paul *Moduli space of irregular connections and Painlevé equations*
10:15 – 11:00	Jean-Pierre Ramis *Dynamics on the Wild Character Varieties of the Painlevé equations*
11:00 – 11:30	Coffee Break
11:30 – 12:15	Marianna Ravara *Nodal separators for codimension one foliations in dimension three*
12:15 – 13:00	David Marín *Topological moduli space for singular germs of holomorphic foliations in the plane*
13:00 – 15:30	Lunch
18:30 – 20:30	SPECIAL SESSION
21:15	Social Dinner

Friday, June 30	
9:30 – 10:15	Xavier Gómez-Mont *Attracting measures in foliations with hyperbolic leaves*
10:15 – 11:00	Laura Ortiz *On the complexity of the holonomy of polynomial perturbations of integrable systems*
11:00 – 11:30	Coffee Break
11:30 – 12:15	Helena Reis *On the dimension of the automorphism group of projective threefolds with* $\mathrm{PIC} = \mathbb{Z}$
12:15 – 13:00	Marcel Nicolau *On the automorphism group of a transversely holomorphic foliation*
13:00 – 15:30	Lunch

ABSTRACTS

Minimal foliations of hyperbolic 3-manifolds

Fernando Alcalde

Foliations by surfaces on compact manifolds generalize compact surfaces in many ways. Poincaré's Uniformization Theorem tells us that *most* compact surfaces are hyperbolic –namely, those with negative Euler characteristic. In a similar way, many (arguably *most*) foliations by surfaces are *hyperbolic* in the sense that they admit a Riemannian metric along the leaves with constant curvature -1. However, there are not many explicit constructions of this kind of foliations. In the talk, we shall show how to construct minimal foliations (with dense leaves) by hyperbolic surfaces on hyperbolic 3-manifolds by using closed 1-forms or branched coverings.

Joint work with Françoise Dal'Bo, Matilde Martínez, and Alberto Verjovsky.

Stratification of the dynamics of real vector fields in dimension three

Clemen Alonso

It is well known that an analytic vector field in the plane with an isolated singular point and a characteristic orbit has a finite sectorial configuration, that is, around the singularity the dynamics splits into a finite union of hyperbolic, parabolic and elliptic sectors. In this talk we generalize this result for analytic three dimensional real vector fields under non degeneracy conditions. This is a joint work with Fernando Sanz Sánchez (University of Valladolid).

The Sard conjecture on Martinet surfaces

André Belotto

This talk concerns an application of resolution of singularities to sub-riemannian geometry. Given a totally nonholonomic distribution of rank two Δ on a three-dimensional manifold M, it is natural to investigate the size of the set of points $\mathcal{X}^x$ that can be reached by singular horizontal paths starting from a same point $x \in M$. In this setting, the Sard conjecture states that $\mathcal{X}^x$ should be a subset of the so-called Martinet surface of 2-dimensional Hausdorff measure zero.

In this seminar, I present a recent work in collaboration with Ludovic Rifford where we show that the conjecture holds whenever the Martinet surface is smooth. Moreover, we address the case of singular real-analytic Martinet surfaces and show that the result holds true under an assumption of non-transversality of the distribution on the singular set of the Martinet surface. Our methods rely on the control of the divergence of vector fields generating the trace of the distribution on the Martinet surface and resolution of singularities.

Jacobian curves of singular foliations

Nuria Corral

The jacobian curve of two singular foliations is the contact curve between both foliations. In this talk, we will present some results concerning properties of the equisingularity type of the jacobian curve in terms of local invariants associated to the foliations.

Ridge of a tangent cone: a forgotten theorem

Vicent Cossart

Let $\mathcal{X}$ be an excellent scheme, we denote by $\mathrm{H}_{\mathcal{X}}$ the modified Hilbert-Samuel function. This function is upper semi-continuous along $\mathcal{X}$ and does not increase for the lexicographical ordering after permissible blowing ups. When $\mathcal{X}$ is embedded in a regular ambient scheme $\mathcal{W}$, for all $x \in \mathcal{X}$, the "stable τ at x"("τ stable de x"), denoted by $\tau_{st}(x)$, is the codimension of the ridge of the tangent cone of $\mathcal{X}$ at x in the tangent cone of $\mathcal{W}$ at x. It is well known that the function

$$\iota \ : \ \mathcal{X} \to \mathbb{N}^{\mathbb{N}} \times -\mathbb{N}, \ x \mapsto (\mathrm{H}_{\mathcal{X}}(x), -\tau_{st}(x)),$$

does not increase for the lexicographical ordering after permissible blowing ups. In this conference, we will show the following:
Forgotten Theorem *The function ι is upper semi-continuous along $\mathcal{X}$.*

We will say in a few words why we call it "Forgotten Theorem".

Local uniformization of codimension one foliations

Miguel Fernández

In this work we prove local uniformization in the sense of Zariski for codimension one singular foliations in any ambient dimension. This is a joint work with F. Cano.

Dimension of the moduli space of a plane branch in $\mathbb{C}^2$

Yohann Genzmer

In 1973, Zariski adressed the problem of the computation of the generic dimension of the moduli space of a plane branch in the complex plane. In this talk, our goal is to present a formula which can be performed by hand, that gives this dimension for any topological class of irreducible curve. The key tool is a study of the logarithmic forms associated to the branch, as introduced by K. Saito in 1980.

Attracting measures in foliations with hyperbolic leaves

Xavier Gómez-Mont

Let $(M, \mathcal{F})$ be a compact manifold with a non-singular foliation with hyperbolic leaves (i.e. having a continous Riemannian metric on the tangent bundle to the foliation of constant negative curvature). Let $T^1_{\mathcal{F}}$ be the unit tangent bundle to the leaves, provided with the foliated geodesic flow

$$\varphi : T^1_{\mathcal{F}} \times \mathbb{R} \to T^1_{\mathcal{F}}$$

Due to the negative curvature assumption, this flow is partially hyperbolic (i.e. it is hyperbolic in the tangential directions of the leaves) but there is no hypothesis on the normal directions to the foliation. Due to this partial hyperboliciy, one may construct a family of measures, called Gibbs-states, on $T^1_{\mathcal{F}}$ which are coming from the distant past.

If one further assumes that the foliation is transversally conformal (as for example for holomorphic foliations in surfaces with hyperbolic leaves) and that there is no transversal conformal measure, then one may use results of Deroin and Kleptsyn to reduce these measures to a convex combination of a finite number of ergodic measures (using foliated Brownian motion). Once one pushes these measures to the original manifold M (forgetting the tangential direction of the foliated geodesic) one obtains a finite number of measures which are entitled to be the true solutions of the foliation (since they are capturing the asymptotic behaviour of the leaves of the foliation). Most interesting is the case when there is only 1 such measure, in case we obtain that our original foliation is an ergodic foliation and we can interpret the measure as being an "attractor".

The main open problem is to extend this differential geometric approach to generic hyperbolic singularities, to obtain a similar result for the generic foliation in $\mathbb{C}P^2$. Results of this type have been obtained by Sibony and his collaborators using plurisubharmonic techniques.

This research has been carried out jointly with Christian Bonatti (Dijon) and Matilde Martinez (Montevideo).

A remark about complex polynomial functions

Lê Dũng Tráng

Let $f : \mathbf{C}^n \to \mathbf{C}$ be a polynomial function. We propose to show that the inverse image of a disk which contains all the atypical values has the homotopy type of a bouquet of spheres.

Realization of formal invariant curve

Olivier Le Gal

Given a real analytic vector field X, singular at the origin of $\mathbb{R}^n$, and C an invariant formal curve for X, we prove that there exists a parametrized real half curve γ, invariant by X, and asymptotic to C. Such a curve can moreover be chosen to be non-oscillating with respect to the analytic category. (Joint work with F. Cano and F. Sanz).

Ensembles pfaffiens

Jean-Marie Lion

Il s'agit de présenter un panorama sur les ensembles pfaffiens avec ou sans le recours aux éclatements chers à Felipe.

Stable manifolds of diffeomorphisms asymptotic to formal curves

Lorena López

We study local dynamics of an analytic diffeomorphism F in $\mathbb{C}^n$. We will show that for any formal invariant curve of F such that the restricted diffeomorphism is either hyperbolic attractive or resonant non-periodic (i.e. for any formal invariant curve compatible with an attracting behaviour) there exists an invariant manifold for F, of some dimension $d \in \{1, ..., n\}$, in which every orbit is asymptotic to the formal curve. This is a joint work with J. Ribón, F. Sanz and L. Vivas.

Neighborhoods of curves in complex surfaces

Frank Loray

Given a smooth irreducible complex compact curve C (i.e. compact Riemann surface) we want to understand and classify embeddings of C in smooth complex surfaces S, i.e. germs of neighborhoods (S, C) up to isomorphisms. After recalling some classical results, we detail the case where C is an elliptic curve having torsion normal bundle in S.

This is a work in progress with Olivier Thom, Frédéric Touzet and Sergey Voronin.

Topological moduli space for singular germs of holomorphic foliations in the plane

David Marín

The aim is to identify the moduli space of topological classes of germs of holomorphic foliations in the plane, once we fix the topological type of its separatrix curve and the semi-local type, given by the holonomy and Camacho-Sad indices.

On the automorphism group of a transversely holomorphic foliation

Marcel Nicolau

We prove that the automorphism group $\mathrm{Aut}(M, \mathcal{F})$ of a transversely holomorphic foliation $\mathcal{F}$ on a compact manifold M is a Fréchet Lie group, and in fact a strong ILH-group in the sense of Omori. Moreover $\mathrm{Aut}(M, \mathcal{F})$ is naturally endowed with a Lie foliation transversely modeled on the complex simply-connected Lie group associated to the Lie algebra of basic holomorphic vector fields $H^0(M, \Theta_{\mathcal{F}})$, which is of finite dimension as it was proved by X. Gómez-Mont.

This is joint work with Laurent Meersseman.

On the complexity of the holonomy of polynomial perturbations of integrable systems

Laura Ortiz

We will consider polynomial perturbations of polynomial integrable systems. We will explain how to provide a universal bound for the complexity of such perturbations, in terms of the unperturbed system.

Resolution of singularities for differential operators

Daniel Panazzolo

In a pioneering work published in 1987, Felipe Cano stablishes some strategies towards the resolution of singularities for three-dimensional vector fields. In this talk, I will describe a new approach to this problem through the use of Kempf's theory of instability in Geometric Invariant Theory. This new approach has the advantage of being valid for arbitrary finite order linear partial differential operators in any ambient dimension.

Zariski equisingularity and Whitney conditions

Adam Parusinski

Varchenko showed that Zariski equisingularity implies topological triviality for families of germs of analytic sets. Using Whitney interpolation, we construct explicit subanalytic trivializations of such families that are analytic on real analytic arcs and, moreover, are analytic with respect to the parameter. Then, given an algebraic set or a germ of an analytic set, we construct its stratification that fibers this set, locally along each stratum, into analytic submanifolds with a strong continuity of tangent spaces, analogous to Verdier's regularity condition (w). This shows Whitney's fibering conjecture.

Moduli space of irregular connections and Painlevé equations

Emmanuel Paul

The Painlevé VI equation appears as an isomonodromic condition for a deformation of connections with regular singular points, and fixed local data, over a base space of parameters: the cross ratios of the singularities. In the irregular cases, we know that the other Painlevé equations describe isomonodromic and iso-Stokes deformations of irregular connections. Nevertheless the geometric picture is not so clear, particularly for the base. The Poisson structures are here a central tool to obtain this description.

Algebraic integration in bounded genus

Jorge Vitorio Pereira

We will show how techniques and ideas from birational geometry can be used to provide information on the Zariski closure of the set of foliations admitting rational first integrals. Based on a joint work with Roberto Svaldi.

Dynamics on the wild character varieties of the Painlevé equations

Jean-Pierre Ramis

We will explain how to associate to an irregular connection on a compact Riemann surface a character variety and describe it for some Painlevé equations.

We will define a wild dynamics on the Okamoto space of initial conditions of the Painlevé equations based on the non-linear Stokes phenomena at infinity. We will translate it into a dynamics on a character variety and we will present some conjectures on this dynamic.

Nodal separators for codimension one foliations in dimension three

Marianna Ravara

The connected union of generically nodal curves in dimension three, which may appear during the reduction of singularities of a codimension one foliation, act as a nodal separator for the leaves near the exceptional divisor. In this work we use the dual graph of the exceptional divisor to study the behavior of these objects - called uninterrupted nodal components - and the consequences they may produce in the structure of the foliation.

On the dimension of the automorphism group of projective three-folds with $\mathrm{PIC} = \mathbb{Z}$

Helena Reis

Recall that the group of holomorphic diffeomorphism $\mathrm{Aut}(M)$ of a compact complex manifold is a Lie group of finite dimension. The Lie algebra of $\mathrm{Aut}(M)$ is then identified with the space of holomorphic vector fields $\mathfrak{X}(M)$ on M. It is natural question to look for bounds on the dimension of $\mathrm{Aut}(M)$ in terms of the dimension of M and some extra-data. In this talk, we will focus on compact projective manifolds of dimension 3 whose Picard group is isomorphic to $\mathbb{Z}$. The main result asserts that the dimension of $\mathrm{Aut}(M)$ in this case cannot exceed 30.

Intersections in holomorphic dynamical systems

Javier Ribón

Let us move a curve containing the origin by the action of a group of origin-preserving biholomorphisms of the plane. We want to characterized the groups such that the set of Milnor numbers, describing the tangency of the iterates of a first given curve with a second fixed curve, is always bounded for any choice of pair of curves. Generalizations of this property in higher dimension are straightforward to define. It is well-known that such a property holds for cyclic groups and has been generalized to other classes of groups by Seigal-Yakovenko, Binyamini and myself. It remains open to know whether it is possible to characterize the groups that hold this uniform intersection property. We will provide a solution of this problem in dimension two.

Characteristic curves of holomorphic foliations

Rudy Rosas

Let $\mathcal{F}$ be holomorphic foliation with singularity at $0 \in \mathbb{C}^2$. A characteristic curve of $\mathcal{F}$ is a continuous one-dimensional curve tending to $0 \in \mathbb{C}^2$ and tangent to $\mathcal{F}$ which is a kind of generalization of a separatrix. We define a notion of desingularization of the set of characteristic curves of $\mathcal{F}$ and show that this desingularization gives us another way of understanding the resolution of singularities of the foliation $\mathcal{F}$. As an application we obtain that the equisingularity class of the set of formal separatrices of $\mathcal{F}$ is invariant by C^∞ conjugations of the foliation. In particular, if $\mathcal{F}$ is a foliation of second type, we obtain that the equisingularity class of $\mathcal{F}$ is also a C^∞ invariant.

Real analytic vector fields with first integral and separatrices

Fernando Sanz

We prove that a germ of analytic vector field at $(\mathbb{R}^3, 0)$ with a non-constant analytic first integral always has a real formal invariant curve. We provide an example which shows that such a vector field does not necessarily have a real analytic invariant curve.

This is a joint work with Rogério Mol.

Khanedani-Suwa variational residues for invariant currents

Marcio Soares

We present a Khanedani-Suwa variational residue type theorem for currents invariant by holomorphic foliations. This is joint work with M. Corrêa and A. Fernández-Pérez.

On the behavior of the multiplicity on points of a variety

Orlando Villamayor

The multiplicity stratifies the points of a variety into subsets of points with the same multiplicity. These sets are locally closed, and the stratum of points with highest multiplicity is a closed set. We will discuss properties of the highest multiplicity locus, and the blow up of the variety along regular centers included in this locus (at equimultiple centers).

The first and simplest context to address the study of the multiplicity is that in which the variety is a hypersurface included in a regular variety. The notion of hypersurface has a natural extension to so called "generalized hypersurfaces"; and we will show that the study of the multiplicity is also simple for those varieties, or schemes, which are generalized hypersurface.

Finally, we show that one can attach to any variety a new scheme which is a generalized hypersurface, so that both schemes have the same stratum of highest multiplicity locus, and furthermore, such identification is preserved by any sequence of blow ups at equimultiple centers.

This latter property has several applications, for example to prove resolution of singularities in characteristic zero by using the multiplicity as an invariant.

Normalization and factorization of linear ordinary differential operators

Sergei Yakovenko

We consider the Weyl-type algebra of ordinary linear differential operators at a singular point and their classification. It is similar but not identical to the gauge classification of systems of first order linear ODEs. In particular, we show how non-Fuchsian (irregular) operators can be decomposed into operators of different Poincare ranks with single eigenvalues, and how this result is connected to the blow-up of singularities of planar analytic curves.

Posters

Multiplicity and finite morphisms

Carlos Abad Reigadas

Let X be a variety over a perfect field k. The multiplicity is an invariant that stratifies X into locally closed sets. X is regular at a point ξ if and only if $\operatorname{mult}(\xi) = 1$. Dade proved that, if

$$X \leftarrow X_1 \leftarrow \cdots \leftarrow X_l \tag{1}$$

is a sequence of blow-ups along closed regular equimultiple centers, then

$$\max \operatorname{mult}(X_l) \leq \max \operatorname{mult}(X).$$

If one can find a sequence like (1) so that $\max \operatorname{mult}(X_l) < \max \operatorname{mult}(X)$, then a resolution of singularities of X can be constructed by iteration of this process.

The study of the multiplicity has been historically linked to that of finite morphisms. Zariski's formula says that, if $\beta : X' \to X$ is a finite and dominant morphism of varieties of generic rank r, then

$$\max \operatorname{mult}(X') \leq r \cdot \max \operatorname{mult}(X). \tag{2}$$

Let $\underline{\operatorname{Max}} \operatorname{mult}(X)$ denote the stratum of maximum multiplicity of X. When the equality holds in (2), $\underline{\operatorname{Max}} \operatorname{mult}(X')$ is homeomorphic to its image in X, which is contained in $\underline{\operatorname{Max}} \operatorname{mult}(X)$. In our work, we study conditions under which

$$\beta\big(\underline{\operatorname{Max}} \operatorname{mult}(X')\big) \cong \underline{\operatorname{Max}} \operatorname{mult}(X),$$

and such that this homeomorphism is preserved by *permissible* blow-ups. These conditions provide a relation between the processes of lowering of the maximum multiplicity of X and X' by blowing up equimultiple centers.

This is a joint work with Ana Bravo and Orlando Villamayor.

Rigidity theorems for germs of foliations with degenerate singularity

Jessica Angélica Juarez Rosas

We consider the class of germs of infinitely differentiable vector fields with an isolated singularity at $(\mathbb{R}^2, \bar{0})$. Two such germs are said to be *orbitally $\mathcal{C}^\infty$ equivalent* if there exists a germ of infinitely differentiable change of coordinates in $(\mathbb{R}^2, \bar{0})$ which maps the phase curves of one germ to the phase curves of the other. They are *orbitally formally equivalent* if there exists a formal change of coordinates in $(\mathbb{R}^2, \bar{0})$ transforming one germ into the other multiplied by some formal power series with non-zero constant term.

Although the orbital $\mathcal{C}^\infty$ equivalence implies the orbital formal equivalence, the converse is in general false, as it is shown by flat perturbations of centers. It is said that *the formal rigidity phenomenon* takes place for classes of germs where the formal equivalence and the $\mathcal{C}^\infty$ equivalence coincide.

In [1] Chen proved that the formal rigidity phenomenon takes place for a class of germs with nonzero linear part at the singular point, namely the class of germs with a *real hyperbolic singularity* (i.e., the eigenvalues of the linear part have nonzero real part).

In [2] we prove the existence of classes of germs of order $n \geqslant 2$ at the origin where the formal rigidity also takes place. This provides a generalization of Chen's theorem for a class of germs of vector fields with degenerate singularity at the origin. In this poster we will outline the proof of this result and compare the used techniques with those used to prove the formal rigidity phenomenon for holomorphic and real analytic foliations ([3], [4]).

References

[1] K.-T. Chen: Equivalence and Decomposition of Vector Fields about an Elementary Critical Point, Amer. J. Math. Vol. 85, 1963, pp. 693-722.

[2] J. A. Jaurez-Rosas and L. Ortiz-Bobadilla: Chen's Theorem for $\mathcal{C}^\infty$ foliations in the real plane with degenerate singularity. In preparation.

[3] S. M. Voronin: Orbital Analytic Equivalence of Degenerate Singular Points of Holomorphic Vector Fields on the Complex Plane, Tr. Mat. Inst. Steklova, Vol. 213, 1996, pp. 30-49.

[4] J. A. Jaurez-Rosas: Real-Formal Orbital Rigidity for Germs of Real Analytic Vector Fields on the Real Plane, J. Dyn. Control Syst. Vol. 23 (1), 2017, pp. 89-109.

Moduli spaces of a family of topologically non quasi-homogeneous functions

Jinan Loubani

We consider a topological class of a germ of complex analytic function in two variables which does not belong to its jacobian ideal. Such a function is not quasi-homogeneous. Each element f in this class induces a germ of foliation ($df = 0$). This poster presents a local result about the moduli spaces of the foliations in this class. More precisely, it gives the dimension of the tangent space to the moduli space, describes the local moduli space and gives local analytic normal forms. It also presents a result regarding the uniqueness of these normal forms.

There are no new index theorems for quadratic vector fields on $\mathbb{C}^2$

Valente Ramírez García Luna

Consider a polynomial vector field of degree $n \geq 2$ on $\mathbb{C}^2$. In the generic case, it has n^2 isolated singularities, and the foliation it defines on $\mathbb{C}P^2$ has an invariant line at infinity with $n+1$ singular points.

Each equilibrium carries two analytic invariants: the spectrum of its linearization matrix. Each singular point at infinity carries one analytic invariant: its Camacho-Sad index. Define the *extended spectra of singularities* to be the collection of these $2n^2 + n + 1$ numbers.

These numbers are constrained by classical *index theorems*: the Euler-Jacobi relations, the Camacho-Sad theorem and the Baum-Bott theorem. A simple dimensional argument shows that, for each fixed degree n, there must exist yet more algebraic relations among these numbers. Not one of these *hidden relations* were, until very recently, known.

In the quadratic case, there is only one such hidden relation. In this poster we will exhibit and explain this last relation. Moreover, we will show that it does not come from an index theorem. In fact, we show that any possible "index-theorem-like equation" can be deduced from the classical index theorems, hence concluding the lack of existence of new index theorems.

This work is a recent collaboration with Yury Kudryashov (Cornell University).

Degree of the exceptional component

Artur Rossini

The space of codimension one foliations and degree two in $\mathbb{P}^3$ has six irreducible components, and from the point of view of enumerative geometry it is interesting to calculate the dimension and degree of such components. In particular, we will present strategies used to determine the degree of the so-called exceptional component of foliations, a 13-dimensional component.

This work is part of my doctoral thesis at UFMG, Belo Horizonte, with my supervisor Israel Vainsencher.

Differential Galois theory and Darboux transformations for integrable systems

Raquel Sánchez Cauce

In [1], we compute the differential Galois group for Schrödinger operator $L - \lambda = -\partial_x^2 + u - \lambda$ for some solitonic solutions u of KdV differential equation. Taking an AKNS system associated to $L - \lambda$, say

$$\Phi_x = U\Phi, \qquad \Phi_t = V\Phi, \tag{$\mathfrak{s}$}$$

as in [1], we can perform suitable Darboux transformations to maintain the Galois group of ($\mathfrak{s}$). Now, we present singular curves associated to the system ($\mathfrak{s}$) and we study the behaviour of its singular locus under these Darboux transformations.

The AKNS systems techniques and the Picard-Vessiot Theory for integrable systems (as appears in Appendix D of [2]) turn out to be essential in our approach to the problem and allow us to state our main result that says that the Galois group of the transformed system is isomorphic to a subgroup of the Galois group of the initial system, proving that, by Darboux transformations, we obtain systems at least as integrable as the initial system. In particular, if the initial system is integrable, the transformed system is integrable too, in complete agreement with Darboux ideas.

This is a joint work with Sonia Jiménez, Juan J. Morales Ruiz and María-Ángeles Zurro.

References

[1] S. Jiménez, J.J. Morales-Ruiz, R. Sánchez-Cauce, M.A. Zurro, Differential Galois theory and Darboux transformations for Integrable Systems, Journal of Geometry and Physics 115 (2017) 75-88.

[2] M. van der Put, M. F. Singer, Galois Theory of Linear Differential Equations, in:Grundlehren der mathematischen Wissenschaften, Volume 328, Springer Verlag, (2003).

Pairs of functions, pairs of foliations

Olivier Thom

I will present some results about the classification of pairs of functions from $(\mathbb{C}^n, 0)$ to $(\mathbb{C}, 0)$ and pairs of codimension 1 foliations on $(\mathbb{C}^n, 0)$ up to diffeomorphism on the right.

More precisely, I will give normal forms for some simple examples, some equivalence criteria for some less simple examples, and I will also present a general result which can be used in a very general setting.

As an example, we have the following :

Proposition. *A generic pair of Morse functions on* $(\mathbb{C}^2, 0)$ *is diffeomorphic to a pair* $(x^2+y^2, u(x)+v(y))$ *where* u *and* v *are Morse functions on* $(\mathbb{C}, 0)$ *uniquely determined up to the action of a group of order* 4.

Here the genericity condition is a condition on the quadratic part of the pair. These results come from the articles [T1] and [T2].

References

[T1] O. Thom. *Classification locale de bifeuilletages holomorphes sur les surfaces complexes*, Bull. Braz. Math. Soc. (N.S.) 47 (2016) 989-1005.

[T2] O. Thom. *Pairs of Morse functions*, 2017, https://hal.archives-ouvertes.fr/hal-01482729.
